**SUPPLEMENTARY MATERIAL**

**A) Over-dispersion test (multiplicative approach).**[27]

The multiplicative approach introduces an over-dispersion factor $\phi$ that inflates the null variance. Suppose we have a sample of "I" units (i.e. I = 82 municipalities for the Palermo Province) that we shall assume (for the present) all to be in-control. $\phi$ may be estimated as follows:

$$\hat{\phi} = \frac{1}{I}\sum_i \frac{(y_i - \theta_0)^2 \rho_i}{g(\theta_0)} = \frac{1}{I}\sum_i z_i^2$$

where $z_i$ is the standardized Pearson residual defined as

$$z_i = \frac{y_i - \theta_0}{\sqrt{\mathbb{V}(Y|\theta_0)}}$$

where $y_i$ is the indicator of interest (SIR of municipality) and $\theta_0$ is the value of target ($SIR_{target}$ = 1). The current control limits can then be inflated by a factor $\sqrt{\hat{\phi}}$ around $\theta_0$. For example, based on the approximate normal control limits, over-dispersed control limits can then be plotted as

$$y_p(\theta_0, \rho) = \theta_0 + z_p\sqrt{\hat{\phi} g(\theta_0)/\rho}$$

where g($\theta_0$) can be choice equal to $\theta_0$, when $y_i$ is a standardised ratio. There is an element of circularity, in that if out-of-control units are included in this estimation process, they will tend to increase the estimate of $\theta_0$, widen the funnel limits and hence make it more difficult to detect the very cases in which we are interested. Therefore. when estimating $\phi$, we may want to 'robustify' the analysis by minimizing the influence of outlying cases that the system is designed to detect. There follows the 'Winsorised' estimation algorithm in which the most extreme cases are shrunk to pre-specified percentiles of the distribution:

1. Rank cases according to their naive Z-scores.
2. Identify $Z_q$ and $Z_{1-q}$, the 100q per cent most extreme top and bottom naive Z-scores, where q might, for example, be 0.1.
3. Set the lowest 100q per cent of Z-scores to $Z_q$, and the highest 100q per cent of Z-scores to $Z_{1-q}$. Denote the resulting set of Z-scores, both those left unchanged and those that have been 'pulled-in', by $Z^w$.
4. Calculate the estimate $\phi$ using $Z^w$, so that

$$\hat{\phi}W = \frac{1}{I}\sum_i Z_i^W(q)^2$$

If there is no true over-dispersion, then $I\phi$ has approximately a $\chi^2$ distribution, with $I$ degrees of freedom, which means that $E(\phi) = 1$, $V(\phi) = 2/I$. Rather than applying the over-dispersion adjustment to all data by default, it may therefore be better to:

1. not assume under-dispersion: i.e. if $\phi < 1$, assume $\phi = 1$;
2. demand a 'statistically significant' $\phi$ before including an adjustment for over-dispersion: i.e. assume $\phi = 1$ unless the estimated $\phi > 1 + 2\sqrt{2/I}$.

In our example $\phi$ (= 13.45647) is more than 10 times the value of $1 + 2\sqrt{2/I}$=1.312348).

**B) R-script developed to detect the greatest cut-off for the winsorization procedure.**

We have written the following r-script in order to automatically detect the "maximum" q value of winsorization:

```
qWINZORING <- function(Znu){
 c<-0
 n<-length(Znu)
 qseq <- seq(from = 0, to = 1, by = 0.001)
 for(q in qseq) {
  c <- c+1
  Zq <- quantile(Znu, probs = q)
  Z1_q <- quantile(Znu, probs = 1-q)
  Ziu <- Znu
  Ziu[Znu<Zq] <- Zq
  Ziu[Znu>Z1_q] <- Z1_q
  phiW <- mean(Ziu^2)
  if(phiW <= 1+2*sqrt(2/n)) break
 }
 qqnorm(Znu); qqline(Znu)
 return(qseq[c-1])
}
```

Where Znu is the vector of Z-scores. This function return the “maximum” q level for the winsorization. This R-function applied to our data returns the value 1, i.e. any $q < 1$ is suitable.

**C) Over-dispersion test (additive approach).**[27]

The random-effects approach assumes that $Y_i$ has expectation $E(Y_i) = \phi_i$ and variance $V(Y_i) = \sigma_i^2$, and that for ‘on-target’ trusts $\phi_i$ is distributed with mean $\phi_0$ and standard deviation $\tau$. Hence the null hypothesis is a distribution rather than a point. $\tau$ can be estimated using a standard ‘method of moments’ estimator

$$\hat{\tau}^2 = \frac{I\hat{\phi} - (I-1)}{\sum_i w_i - \sum_k w_i^2 / \sum_i w_i}$$

where $w_i = 1/\sigma_i^2$, and $\phi$ is the test for heterogeneity: if $\phi < (I - 1)/I$, then $\tau^2$ is set to 0 and complete homogeneity is assumed. The funnel plot boundaries are then given by

$$\theta_0 \pm z_p \sqrt{\mathbb{V}(Y|\theta_0, \rho) + \tau^2}$$

In our data, $\tau^2$ results equal to 0.0007151463.